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Determination of the yield strength through bending tests and finite element analysis

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Abstract

The yield strength is typically determined by performing tensile tests, as material non-linearities preclude accurate estimation in other load cases. Conversely, the objective of this project is to obtain these material parameters from the force-deformation curve of a four-point bending test. To this end, an iterative optimization process was implemented using the finite element method and the programming language R. The SDAR-algorithm is used to determine the Young's modulus. The estimation of the yield strength and the constant n is achieved through the Ramberg-Osgood-Equation, a model that successfully delineates the stress-strain curve with a mere three parameters. Synthetic test data was used to determine the accuracy of yield strength determination, yielding a relative error of 0.2 %.

I. Introduction

To select a material that is satisfying the design specifications of a mechanical construction, it is important to know the material parameters as there can be various differences, even within an alloy. Some of the most important parameters are the Young's modulus E, the yield strength σ_y and the Poisson's ratio ν. These parameters provide information about the mechanical properties of a material, which is important when forces or loads are applied on an object to ensure the functionality and load capacity [1]. For medical devices, this means that the material used needs to be tested in terms of biocompatibility, stability and endurance.
Different testing methods may be used for this depending on the applicable standard. Often the tensile test is performed to determine the Young's modulus, the yield strength and the tensile strength from the stress-strain curve [2]. Especially the latter is often used to predict the behaviour by Finite Element Analysis (FEA) [3]. As in tensile testing, stress is homogeneous in cross-sections perpendicular to the test direction, this method is preferable for an accurate estimation of yield strength.
In this project, however, the four-point bending test is used to determine the properties of a material with continuous yielding by numerically compensating for the effect of material non-linearity in the bending load case.

II. Methods and materials

Figure 1 illustrates a conventional engineering stress-strain curve, which results from a tensile test. The slope of the elastic region is known as Young's modulus. As the linearity of stress and strain ends, the yield strength is reached. Beyond this point, the material deforms plastically. If the materials do not show a pronounced yield point but continuous yielding as in Figure 1, then the 0.2 % (defined as $R_{p0.2}$ [4]) offset method is applied to find the yield point [5].

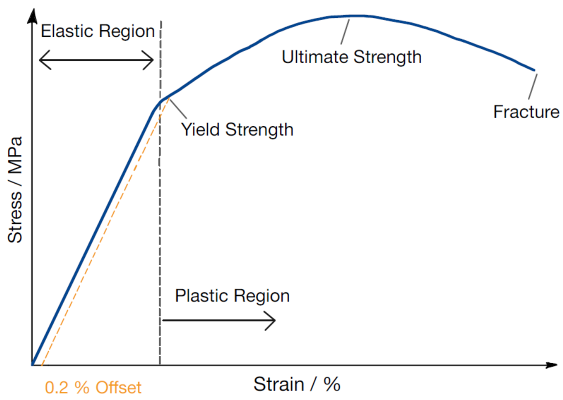

Figure 1: Conventional stress-strain curve

Especially, the differentiation between true stress and strain and engineering stress and strain is essential for the outcome of this project. Ansys (Version 2024 R2, ANSYS, Inc., Canonsburg PA, USA) requires the true value for the Young's modulus. Conversely, the engineering values are extracted after the simulation. A conversion between those two is possible.

For the simulations a *synthetic material model* is generated matching the requirements of Ansys. It contains the true Young's modulus, the Poisson's ratio of 0.33 and a table of the plastic strain-stress data.

II.I. Material model

A synthetic material matching the properties of an aluminum alloy Al 6060 T66 was created by modelling the stress-strain data in R (Version 4.3.2, R Foundation for Statistical Computing, Vienna, Austria) with a Young's modulus of 69 000 MPa, a yield strength of 160 MPa, a tensile strength of 215 MPa and a maximal strain of 0.08 [6]. The stress-strain curve can be completely described by three material parameters using the Ramberg-Osgood-Equation [3], therefore the material is evaluated in regard of the Young's modulus E, the yield strength σ_y and the constant n. The stress-strain curve of this material is labelled as *synthetic material model*. Based on the synthetic material a material model for FEA is generated to simulate a tensile test and a four-point bending test. The simulated tensile test validates the material model and serves as an estimation of error margins occurring due to FEA. For the simulated four-point bending test, a correction of the stress-strain data and optimization strategy is required to estimate material parameters equivalent to tensile testing. The aim is to estimate the yield strength from a bending test with an error similar to the errors found in the simulated tensile test.

II.II. Tensile test

The conventional approach to determine the stress-strain curve is to perform a tensile test [1]. For this test a sample with the dimensions length l, width b and height t is clamped with both ends into the test machine. One end is rigidly fixed, on the other end a force is applied, see Figure 2 [7].

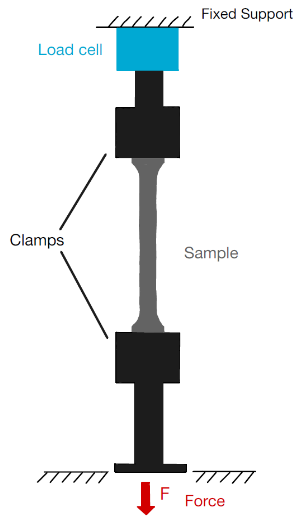

Figure 2: Schematic representation of setup for a tensile test

This test is modelled numerically in Ansys with a maximal deformation of 15 mm. The results are evaluated in R. To determine the Young's modulus the Slope Determination by Analysis of Residuals-algorithm (SDAR-algorithm) is used [8].

Equation (1) describes the stress-strain curve by three parameters. Here, the 0.2 % offset method integrated as one secant. This equation, called Ramberg-Osgood-Equation, is used here for a robust determination of the yield strength and the constant n [3].

$$\epsilon = \frac{\sigma}{E} + 0.002 * \left(\frac{\sigma}{\sigma_y}\right)^n \tag{1}$$

II.III. Four-point bending test

In Figure 3 the setup for a four-point bending test is shown. Two loads are applied on the top of the sample in distance a to each other. The sample lays on two support rollers with distance h from the load rollers.

This test setup is modelled in Ansys and performed with a maximum displacement of 30 mm, called *Bending test*. From the simulation results of the *Bending test* the force reaction and the directional deformation are extracted and visualized in a force-deformation curve, see Figure 4.

By using the dimensions of the sample, the distances of the setup and extracted information about the force F

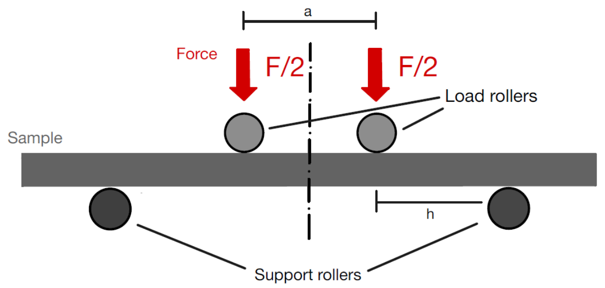

Figure 3: Schematic representation of setup for a four-point bending test

and deformation w at the point of the force, the bending stress σ_b and strain ϵ are calculated with (2) and (3), respectively [9].

$$\sigma_b = \frac{3 * F * h}{b * t^2} \quad (2)$$

$$\epsilon = \frac{w * t}{(\frac{2}{3}h + a) * h} \quad (3)$$

However, (2) is only valid for the linear region. Kato et al. investigated the possibility to estimate the stress-strain curve from a bending test. Here, the assumption is made that tensile ϵ_1 and compressive ϵ_2 strain are identical, see (4) [10].

$$\epsilon = \epsilon_1 = \epsilon_2 \quad (4)$$

Based on the force reaction and determined strain, the outmost fiber stress σ is calculated with (5) [10].

$$\sigma = \frac{h}{b * t^2 * \frac{d\epsilon}{dF}} * (F * 2\frac{d\epsilon}{dF} + \epsilon) \quad (5)$$

With the strain and the newly calculated stress, the material parameters (Young's modulus, yield strength and constant n) are determined with the SDAR-algorithm and the Ramberg-Osgood-Equation. The material parameters estimated from this first simulated bending test cannot accurately match the tensile material parameters due to the effect of material-non-linearities. Thus an iterative optimization is conducted starting with a new material model created from this initial estimate and a simulation in the same four-point bending test, labelled as *Simulation 1*.

The resulting data is used to calculate the stress after Kato et al., and is evaluated concerning the three material parameters. First, the optimization of the Young's modulus is performed by determining the quotient of the engineering Young's modulus from *Bending test* and the engineering Young's modulus from *Simulation 1*.

A new material model for *Simulation 2* is generated with the quotient used as a scaling factor for the Young's modulus determined from *Bending test*. Here, only one iteration is necessary to estimate the Young's modulus.

Equally to *Bending test* and *Simulation 1* the material parameters are determined and in the next step the yield strength is optimized. The criteria for this parameter is the force at the point of deformation where there is 0.2 % strain. Analogous to the Young's modulus ratio, the ratio of this criteria between *Simulation 2* and *Bending test* is calculated and applied to scale the yield strength from *Bending test*.

In order to optimize the third parameter, a new material model is generated using the scaled yield strength and simulated in Ansys. The criteria of optimization for the constant n is the gradient at 0.2 % strain.

Finally, a new material model is generated with the three optimized material parameters and simulated as *Simulation 4*. The force-deformation curves for all simulations are shown in Figure 4.

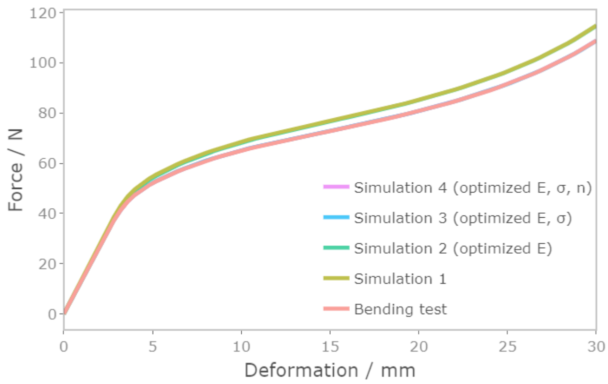

Figure 4: Force-deformation curve from the different optimization steps. *Bending test*, *Simulation 3* and *Simulation 4* are visually nearly identical, equivalently for *Simulation 1* and *Simulation 2*.

III. Results and discussion

Figure 5 illustrates the behaviour of the three stress-strain curves derived from the engineering data. The blue curve represents the synthetic material model, the red curve describes the behaviour of the four-point bending test and green illustrates the curve from the simulated tensile test.

Table 1 shows the initial material parameters and the results obtained by the algorithm for the tensile test and the four-point bending test.

The relative error from the results of the tensile and four-point bending test to the initial parameters are shown in Table 2.

The obtained error values of the tensile test give guidance how exact the material parameters can be determined. The constant n is numerically costly to optimize further: the error only changes from 0.78% with 2000 data points to 0.73% with 10000 data points.

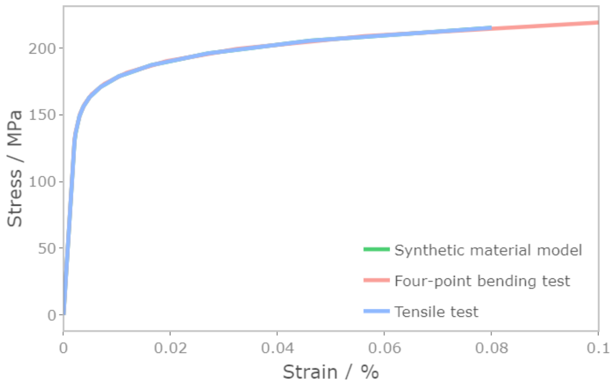

Figure 5: Stress-strain curve from the synthetic material, the simulated tensile test and the four-point bending test (after optimization). The synthetic material model is displayed beneath the curve from the tensile test.

Table 1: Initial engineering parameters and estimated engineering material parameters from tensile and four-point bending test.

Origin	Material Property	Value
Initial	Young's modulus	69.017 GPa
	Yield strength	160.252 MPa
	Constant n	12.357
Tensile	Young's modulus	69.212 GPa
	Yield strength	160.258 MPa
	Constant n	12.269
Bending	Young's modulus	69.013 GPa
	Yield strength	160.299 MPa
	Constant n	12.443

Table 2: Relative error from the tensile and four-point bending test in regard to the initial parameters.

Origin	Material Property	Relative Error
Tensile	Young's modulus	0.28 %
	Yield strength	0.00 %
	Constant n	-0.72 %
Bending	Young's modulus	0.02 %
	Yield strength	0.20 %
	Constant n	0.78 %

Finally, it has to be mentioned that the assumptions in this report only apply to prismatic specimen. Different geometries may need adaptations to the algorithm.

IV. Conclusion

In conclusion, the aim of this project was achieved. The three material parameters Young's modulus, yield strength and the constant n were determined within an error margin similar to the simulation error of a tensile test and the stress-strain curve was determined accordingly.

Additionally, this algorithm is only applicable for ductile materials and has not yet been used for other materials than the one in this report. Also the geometry for the beam is important in some calculations. So before this algorithm can be used with beams of another geometry, changes in the code need to be made. Lastly, although the results are encouraging, further validation is required through practical studies.

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Author's statement

Conflict of interest: Authors state no conflict of interest.

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