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Temporal Priors from Lung Models in EIT Image Sequence Reconstruction

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Abstract: Discrete Cosine Transfer (DCT) based EIT reconstruction has improved EIT image reconstruction by the addition of prior patient information. In this paper this is furthered by introducing prior patient data into the regularisation term. Three different regularisation terms were used, Tikhonov and two regularisation terms containing DCT coefficients (FDRT and DDRT). The regularisation terms were used for reconstruction of two different lung patterns. The results showed that the DDRT performed better than the FDRT for both lung patterns and both out performed Tikhonov for the gradient lung pattern, showing the potential of introducing prior data into the regularisation term.

I. Introduction

Electrical Impedance Tomography (EIT) is an imaging technique most commonly used for imaging the lung, which uses electrical current and the measurement of induced voltages to reconstruct conductivity variations inside the thorax. A DCT based algorithm which has been used for EIT image reconstruction in multiple papers performs reconstruction based on conductivity changes within the lungs and allows input of prior information into the EIT reconstruction, personalising the image [1][2].

The aim of this contribution is to implement a DCT based EIT reconstruction method, with DCT coefficients as priors in the regularisation matrix allowing further personalisation of reconstruction. The DCT coefficients will be integrated into the regularisation term using two different methods reconstructed for to two different reconstruction patterns.

II. Material and methods

II.I. DCT EIT Reconstruction

In the DCT based EIT reconstruction method, the internal conductivity distribution, $\mathbf{x} = \sigma - \sigma^{\text{baseline}}$, is non-linearly related to the difference in measured voltage which is represented by $\mathbf{y} = \mathbf{v} - \mathbf{v}^{\text{baseline}}$. To calculate the conductivity change $\hat{\mathbf{x}}$, equation (1) is used.

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \left\{ \|\mathrm{F}(\mathbf{x}) - \mathbf{y}\|_2^2 + \lambda^2 \|\mathbf{R}\mathbf{x}\|_{\mathrm{j}}^{\mathrm{j}} \right\} \qquad (1)$$

Where $\mathbf{F}(\mathrm{x})$ is the non-linear forward model which maps the conductivity changes $\mathbf{x}$, to measured voltage at the electrodes, $\mathbf{y}$ and j is the corresponding frequencies. In EIT it is assumed that the conductivity changes are very small and therefore the function $\mathbf{F}(\mathrm{x})$ can be linearised around a reference conductivity (σ^{baseline}) as the function $\mathbf{F}(\mathrm{x}) \approx \mathbf{J}\mathbf{x}$. The linearised equation for conductivity distribution is given by:

$$\hat{\mathbf{x}} = (\mathbf{J}^{\mathrm{T}}\mathbf{J} + \lambda^2 \mathbf{R}^{\mathrm{T}}\mathbf{R})^{-1}\, \mathbf{J}^{\mathrm{T}}\mathbf{y} = \mathbf{B}\mathbf{y} \qquad (2)$$

Where $\mathbf{J}$ is the Jacobian matrix and $\mathbf{B}$ is the reconstruction matrix which maps the voltage change to changes in the conductivity distribution.

A subset of DCT coefficients is introduced to modify the Jacobian matrix. The subset consists of a sum of cosine functions the formulation of which is given by:

$$\mathrm{V}_{\mathrm{p,q}} = \sum_{\mathrm{m}=0}^{\mathrm{M}-1} \sum_{\mathrm{n}=0}^{\mathrm{N}-1} \mathbf{A}_{\mathrm{m,n}} \cdot \mathrm{D}(\mathrm{p},\mathrm{q})_{\mathrm{m,n}} \qquad (3)$$

Where A is a two-dimensional image with M rows and N columns, and p and q are the frequencies of the cosine functions.

Matrix $\mathbf{C}(\mathrm{p},\mathrm{q})_{\mathrm{m,n}}$ is produced by multiplying the cosine function with the binary anatomical image, $\mathbf{C}(\mathrm{p},\mathrm{q}) = \mathrm{P}_{\mathrm{m,n}} \cdot \mathrm{D}(\mathrm{p},\mathrm{q})$, allowing the columns of the subset to be calculated as $\mathbf{K}_{\mathrm{j}} = \mathrm{T}(\mathbf{C}(\mathrm{p},\mathrm{q}))$, where T is the mapping function which assigns a corresponding Finite Element Method (FEM) element to each pixel of $\mathbf{C}(\mathrm{p},\mathrm{q})_{\mathrm{m,n}}$. $\mathbf{K}_{\mathrm{j}}$ can then be used to update the Jacobian matrix $\mathbf{J}$ for the DCT reconstruction to give $\mathbf{J}_{\mathrm{DCT}} = \mathbf{J}^{\mathrm{n_{meas} \times n_{elem}}} \mathbf{S}^{\mathrm{n_{elem} \times n_{DCT}}}$, containing $\mathrm{n_{meas}} \times \mathrm{n_{DCT}}$ elements. Introducing this to the inverse problem gives the change of DCT coefficients $\hat{\mathbf{x}}_{\mathrm{DCT}}$ as:

$$\hat{\mathbf{x}}_{\mathrm{DCT}} = \left(\mathbf{J}_{\mathrm{DCT}}^{\mathbf{T}}\mathbf{J}_{\mathrm{DCT}} + \lambda^2 \mathbf{R}^{\mathbf{T}}\mathbf{R}\right)^{-1} \mathbf{J}_{\mathrm{DCT}}^{\mathbf{T}}\mathbf{y} = \mathbf{B}_{\mathrm{DCT}}\mathbf{y} \qquad (4)$$

This change in DCT coefficients can be used to reconstruct the image, $\mathbf{H}$, as shown in the equation below.

$$\mathbf{H} = \sum_{\mathrm{p}=0}^{\mathrm{n_{X_{DCT}}}} \sum_{\mathrm{q}=0}^{\mathrm{n_{Y_{DCT}}}} \mathbf{C}(\mathrm{p},\mathrm{q}) \cdot \hat{\mathrm{x}}_{\mathrm{DCT,j}} \qquad (5)$$

Here, j represents the corresponding p and q for each DCT coefficient, in these simulations these values were set to $\mathrm{p} = 1$ and $\mathrm{q} = 0$.

II.II Regularisation Term

The regularisation term provides prior information and stabilises the solution. Here, DCT coefficients are applied to the regularisation matrix in two ways. The first method was the Fixed DCT Regularisation Term (FDRT), where the calculated DCT coefficients are directly applied to **R**, the equation is shown below [1].

$$\mathbf{R}_{\mathrm{DCT}} = \left(diag\left(\frac{1}{\mathbf{h}}\right)\right)^2 \tag{6}$$

Where **h** is a vector of the expected DCT coefficients from a prior image such as a CT scan. A matrix of the first 15x15 DCT coefficients values was selected and sorted in column order to form vector **h**, and $\mathbf{R}_{\mathrm{DCT}}$ was calculated. The second method used was the Difference DCT Regularisation Term (DDRT). This method uses the difference between the DCT coefficients from the prior image and the voltage measurements.

$$\mathbf{R}_{\mathrm{DCT}} = \left(\frac{1}{\mathrm{DCT}_{\mathrm{IMG}}} - \mathrm{DCT}_{\mathrm{REC}}\right) \tag{7}$$

$\mathrm{DCT}_{\mathrm{IMG}}$ is a matrix of the DCT coefficients calculated directly from the CT image of the lungs and $\mathrm{DCT}_{\mathrm{REC}}$ is a matrix of the DCT coefficients calculated from the voltage measurements. To produce the $\mathrm{DCT}_{\mathrm{IMG}}$ matrix the FDRT method was used and inverted to ensure the same order of magnitude. To produce the $\mathrm{DCT}_{\mathrm{REC}}$ matrix, Tikhonov regularisation was used to calculate a base voltage measurement DCT coefficient matrix. This base DCT matrix was then substituted into equation 6 to produce a new regularisation term, $\mathbf{R}_{\mathrm{DCT}}$. $\mathbf{R}_{\mathrm{DCT}}$ was then used to calculate a new $\mathrm{DCT}_{\mathrm{REC}}$ matrix. This process was repeated until a $\mathrm{DCT}_{\mathrm{REC}}$ matrix which gives a stable regularisation term, $\mathbf{R}_{\mathrm{DCT}}$ which minimizes difference from the prior.

III. Results and discussion

The reconstructions produced and a graph of the quantitative analysis values are shown below.

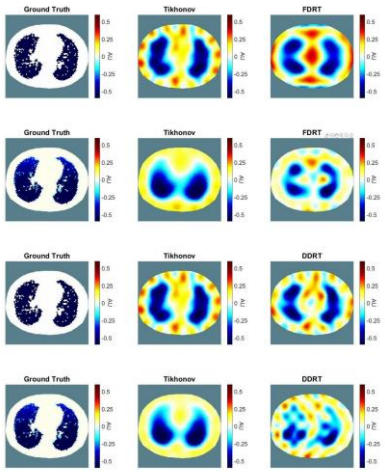

Figure 1. Row 1: FDRT Reconstruction using the homogeneous lung pattern. Row 2: FDRT Reconstruction using the gradient lung pattern. Row 3: DDRT Reconstruction using the homogeneous lung pattern. Row 4: DDRT Reconstruction using the gradient lung pattern.

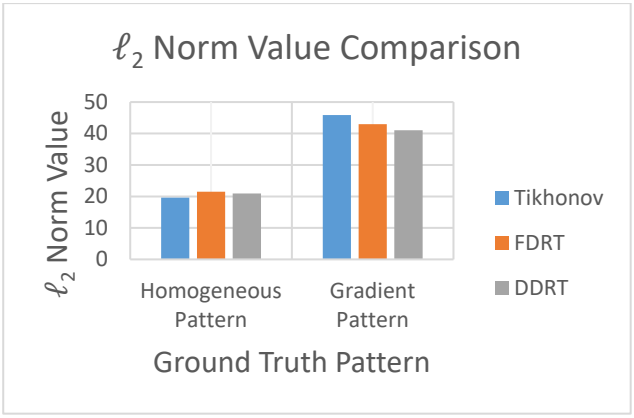

Figure 2. Graph of the ℓ_2 norm values.

From comparison of the visual analysis and the ℓ_2 norm values for each images found in figure 2 the following results can be collected.

For the homogeneous lung pattern, the Tikhonov reconstruction is the most accurate to the ground truth, however there is not a large difference in ℓ_2 norm values between any of the reconstructions. Tikhonov regularization is designed for reconstruction of a homogeneous lung pattern, so this is still a positive result. Of the two regularization techniques introduced in this paper the DDRT gave a more accurate reconstruction than the FDRT.

The ℓ_2 norm values of the gradient lung pattern are much higher than those of the homogeneous lung pattern meaning all of the reconstructions of the gradient pattern are significantly less accurate than the homogeneous lung pattern reconstructions for all regularization terms, although they are still reasonable images. However, both the DDRT and the FDRT reconstructions produce more accurate reconstructions than the Tikhonov regularization for this lung pattern. Again, the DDRT produced a more accurate reconstruction than the FDRT for both lung patterns showing it is a more successful method of introducing the DCT coefficients into the regularization term.

IV. Conclusions

Overall, these new regularisation techniques were successfully implemented and produced images of reasonable quality, their results show there is potential for clinical use. Further investigation is required to confirm if these methods of introducing prior information into EIT reconstruction could be utilised as a standard imaging modality in a clinical setting however, their potential can be seen. In particular, the promising results from the DDRT method are an indication of the potential impact this technique could have in the future, with further investigation.

References

[1] Schullcke, B. et al., 2016. Structural-functional lung imaging using a combined CT-EIT and a Discrete Cosine Transformation reconstruction method. *Scientific Reports*, 6(25951).

[2] Chen, R. & Moeller, K., 2021. Redistribution Index – Detection of an Outdated Prior Information in the Discrete Cosine Transformation-based EIT. *2021 43rd Annual International Conference of the IEEE Engineering in Medicine & Biology Society (EMBC)*, pp. 3693-3696.