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A simple mathematical lung model of Chronic Obstructive Pulmonary Disease (COPD)

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Abstract: This article proposes a one-compartment COPD model and demonstrates the identification of the model parameters using one illustrative patient data set. A linear lung mechanics model is extended by a volume-dependent airway resistance to mimic the dynamic behavior during expiration in typical COPD patients. The model parameters are analytically identified with a focus on the switching time constants of the lung mechanics. The fitted model parameters are in a pathophysiological plausible range. The results show the ability of this simple model to represent the overall lung mechanics characteristic of COPD disease and a feasible first approach for the identification of the model parameters.

I. Introduction

Chronic obstructive pulmonary disease (COPD) is one of the leading causes of death according to the World Health Organization [1]. Contrary to its spread, it is often underdiagnosed and under-treated, in particular in the early stage of the disease [2]. Because COPD is in general leading to an expiratory flow limitation, the disease is diagnosed as a reduced ratio of forced expiratory volume in 1 second (FEV1) to the forced vital capacity (FVC) using spirometry. Also, the expiratory part of the flow-volume curve contains typically a 'kink' compared to the physiological curve due to the expiratory airflow limitation in COPD [3]. According to the Global Initiative for Chronic Obstructive Lung Disease (GOLD), the understanding of lung mechanics is recommended as an additional investigation when the symptoms and the measured airway obstruction do not align in COPD patients. To understand lung mechanics, mathematical models can be used [3]. In the literature, there are existing lung mechanics COPD models [[4], [5]]. However, due to the number of model parameters, they tend to be complex and can be elaborate to identify. Therefore, the main goal of this work is to present a simple mathematical lung mechanics model of COPD and identify the needed model parameters on an illustrative data set of a spontaneously breathing COPD patient [6].

II. Material and methods

Firstly, the mathematical lung mechanics model of COPD is described. Then, the identification of the model parameters using an illustrative data set is covered.

II.I. A mathematical model of COPD

The simple COPD model is derived based on the one-compartment lung mechanics model by Bates et al. [3] for pressure-controlled ventilation. It has been extended by a volume-dependent airway resistance $R(V)$ and a pressure source $P_{mus}(t)$ for the muscle pressure generated by the spontaneous breathing patient

$$P_{aw}(t) = R(V) \cdot \dot{V}(t) + E \cdot V(t) - P_{mus}(t). \qquad (1)$$

E is describing the lung elastance, $V(t)$ the lung volume, and $\dot{V}(t)$ the flow into the lung. $P_{aw}(t)$ is the airway pressure generated by the ventilator. During expiration, if the volume falls below the critical volume V_{crit}, the resistance is increased from R_1 to R_2. At the beginning of the inspiration, $R(V)$ is switched back to R_1. This is to simulate the typical partial collapse of the lower airways in COPD patients during expiration. In Figure 1 a resulting volume curve $V(t)$ from the model is illustrated.

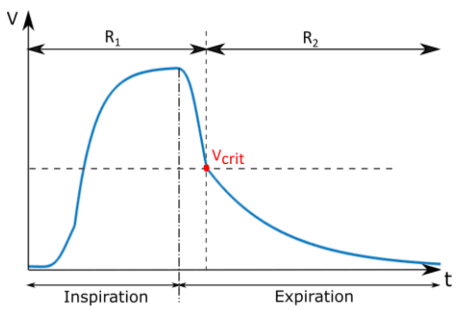

Figure 1: Graphical representation of a resulting volume curve using the proposed COPD model and marking of the changeover point at V_{crit} (modified from [7]).

II.II. Parameter identification

To fully identify the proposed model, the parameters listed in Table 1 need to be fitted to patient data. One illustrative patient data set from Graßhoff et al. [4] is used in this work. The general approach is to identify analytically reasonable values for the model parameters. First, the elastance E is computed via

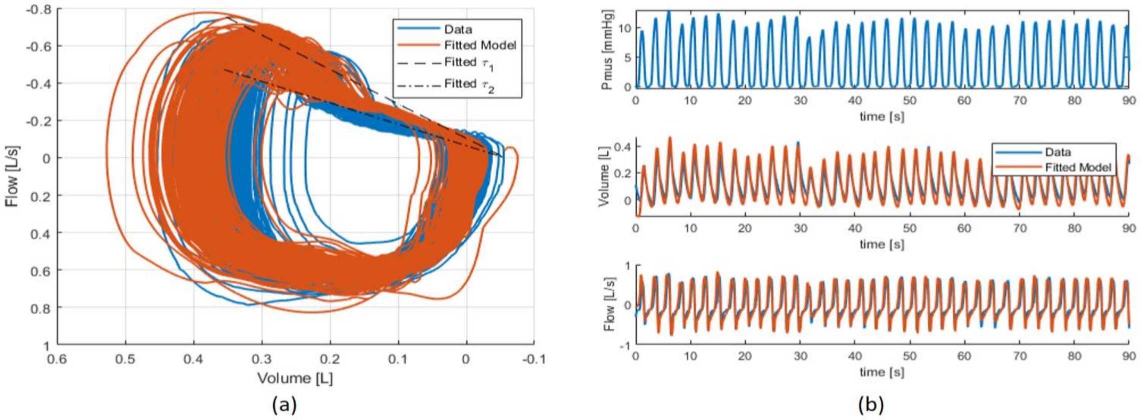

Figure 2: (a) Flow-Volume curve of the patient data [4] and the fitted COPD model. Fitted time constants: straight line based on $V = -\tau \cdot \dot{V} + V_0$. (b) Time course of P_{mus}, volume and flow of the patient data [4]and the fitted COPD model.

$$E = \frac{\Delta P}{\Delta V}. \quad (2)$$

ΔP and ΔV are derived from the volume and muscle pressure curve respectively of the patient data by determining the difference of the end-inspiratory and end-expiratory values. Then, the initial critical volume V_{crit} is calculated by detecting the 'kink' for each breath in the volume curve during expiration (as illustrated in Figure 1) and then computing the mean over all breaths. Next, the time constants τ_1 and τ_2 before and after the switching point during expiration are calculated. To do so, a straight line of the form $V = -\tau \cdot \dot{V} + V_0$ is fitted into the flow-volume curve for the section after the collapse of the lower airways to determine τ_2 and V_0 (see Figure 2 (a), dashed-dotted line). Using the identified V_0, τ_1 is calculated via

$$\tau_1 = \frac{V(t) - V_0}{\dot{V}(t)} \quad (3)$$

for the section before the collapse (see Figure 2 (a), dashed line). As a last step, the resulting airway resistance R_i is calculated

$$R_i = \tau_i \cdot E \text{ with } i = 1,2. \quad (4)$$

III. Results and discussion

In Table 1 the results of the analytical computation of the parameters of the COPD model are listed.

Table 1: Identified parameters of the COPD model.

COPD model parameter		Results
Name [Unit]	**Acronym**	**Value**
Elastance [L/mmHg]	E	14.89
Resistance 1 [mmHg/L/s]	R_1	8.09
Resistance 2 [mmHg/L/s]	R_2	13.01
Time Constant 1 [s]	τ_1	0.54
Time Constant 2 [s]	τ_2	0.87
Critical Volume [L]	V_{crit}	0.198

The resulting simulated flow-volume curve and the time course of the volume and flow compared to the patient data is shown in Figure 2. It can be seen, that the model is mimicking the typical expiratory 'kink' in COPD patients. In addition, both the switching point V_{crit} and the time constants τ_1 and τ_2 seem to be in pathophysiological range and may match the data set. In general, the comparison of the flow-volume curves shows that especially the expiratory behaviors seem to match well (see Figure 2).

IV. Conclusions

In this work, a simple mathematical model of COPD and a first approach for parameter identification is proposed to simulate the effects of changed lung mechanics in COPD patients. Compared to more complex model this simple COPD model provides the advantage, that the parameter identification is solvable. The fitting can be improved in further work, for example by using a fuzzy transition of the model parameters and by using more data.

Author's statement

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