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Modelling of heat diffusion for temperature-controlled retinal photocoagulation

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Abstract: Recent studies demonstrate the potential of temperature controlled retinal photocoagulation. The uniformity of the coagulated spots can be achieved by means of open-loop and closed-loop control. However, no safety-relevant parameters such as maximum laser power or maximum peak temperature have been integrated into the controller design yet. In model predictive control, these constraints can explicitly be taken into account in the controller design. We propose a new concept towards a safer treatment, which includes the consideration of the underlying heat diffusion equation for estimation and control.

I. Introduction

Laser photocoagulation is one of the most frequently used treatment approaches for retinal diseases such as diabetic macular edema or retinopathy. However, it is still a painful and time-consuming treatment as the physician needs to set the laser power and exposure time manually for every irradiated spot due to the variance of retinal pigmentation and light scattering within the eye. Temperature controlled laser photocoagulation is a promising approach to obtain a uniform outcome irrespective of pigmentation of the spot while reducing the duration of treatment. For control, it is necessary to monitor the retinal temperature in real-time. The temperature of the irradiated sites can be determined using a non-invasive optoacoustic method [1].

We can distinguish three main existing approaches to control the temperature based on optoacoustic temperature measurements. In [2] the irradiation stops as soon as the target temperature is reached, which leads to a variable exposure time depending on absorption. To ensure a fixed irradiation time at every spot a three-phase approach has been proposed in [3]. In a so-called probe phase, the absorption is indirectly determined by the temperature rise. Based on the temperature reached in the probe phase, the laser power for the treatment is calculated. The closed loop control in [4] is based on a first order system. A robust H_∞-control and a robust adaptive control scheme were designed. However, the adaptive scheme did not show noticeable improvement in comparison to the robust control. One of the main drawbacks of the applied controllers is the lack of guarantees. Overheating of the tissue cannot be excluded with certainty. In addition, limitations of the laser power cannot be taken into account, which can cause undesirable effects when limits are activated.

In this paper, we present a concept that aims to overcome the disadvantages of the previous controller designs. Based on the partial differential equation (PDE) of heat diffusion, a high dimensional state-space model is developed. This state-space model or suitably reduced order models thereof can serve as the basis for development of advanced control methods such as model predictive control (MPC) [5], which can handle efficiently system constraints. Furthermore, observers can be designed which provide information about the temperature distribution and avoid approximate conversion functions for obtaining the peak values of the temperature, as currently done in the literature [2] - [4].

II. Concept

In order to use the heat diffusion equation in control and observer design, a common approach is to first spatially discretize the equation. The result is a high dimensional model, which can be reduced with various model order reduction techniques. Our focus will be on techniques that allow an evaluation of the model reduction error. The model reduction error can be seen as a bounded model uncertainty.

We design an observer to reconstruct the spatial and temporal temperature profile from the laser power and the optoacoustic temperature (T_{OA}) measurements. The absorption mainly takes place in the retinal pigment epithelium (RPE). Hence, the peak temperature at the center of the RPE is of particular interest. As the peak temperature can be significantly higher than T_{OA}, damage to the tissue can occur if the average temperature, i.e., T_{OA}, is used as a control parameter. A state estimation based on the optoacoustic pressure signal, which is used to determine T_{OA}, is also conceivable. In this case, the inclusion of the photoelastic wave equation [7] in the observer design is essential.

Furthermore, we will design controllers that can take especially safety-critical issues such as maximum allowed peak temperature and restrictions on the laser power into account. Therefore, one main focus will be on model predictive controllers. However, also different controllers such as, e.g., adaptive controllers, will be investigated. In addition to the aforementioned advantage, in model based control design the distributed temperature profile instead of just the peak temperature can be incorporated and also constraints on the distributed temperature profile instead of the peak temperature only can be considered. If the order of the system is reduced, the reduction error can be considered in a robust MPC framework, which can further enhance the safety and control performance.

III. Modelling

During photocoagulation, the fundus of the eye is heated according to the heat diffusion equation

$$\rho C_p \frac{\partial \Delta T(r,t)}{\partial t} - \nabla[k\, \nabla\, \Delta T(\boldsymbol{r},t)] = Q(\boldsymbol{r},t)\,. \qquad (1)$$

The tissue parameters density ρ, heat capacity C_p and thermal conductivity k are assumed to be equal to those of water. We consider the light-tissue interaction as a heat source

$$Q(\boldsymbol{r},t) = \frac{P_L}{\pi R^2}\, \mu_a(z) e^{-\int_0^z \mu_a(\zeta) d\zeta}\,, \qquad (2)$$

with laser power P_L, absorption coefficient μ_a and radius R of the irradiated area. The geometry of our model is a cylinder with a circular irradiated area. The cylinder consists of several layers of the eye fundus. We model a rabbit's eye fundus, whereby the layer thicknesses and the absorption coefficient of the individual layers are selected according to [3]. Due to the symmetry along the z-axis, the 3D model can be reduced to a 2D model. Equation (1) is spatially discretized using central finite differences and temporally using the Euler method. After the spatial discretization, the resulting discrete state-space model consists of 10248 states depending on the selected distances of the finite differences.

The determination of the spot temperature using the optoacoustic method provides an averaged volume temperature increase

$$\Delta T_{volume}(t) = \frac{1}{\pi R^2} \int_V \mu_a(z) \Delta T(\boldsymbol{r},t) e^{-\int_0^z \mu_a(\zeta) d\zeta} d\boldsymbol{r}\,, (3)$$

where ΔT_{volume} serves as the systems output. Thus (3) needs to be discretized according to the system states.

We consider here the balanced truncation approach [6] for model order reduction. Fig. 1 shows the calculated peak temperature during irradiation with 20 mW for 30 ms and cooling for 20 ms. Fig. 2 shows the error between reduced models and the high-dimensional model. The maximum error of the peak temperature is less than 0.1 K when reducing the model from more than 10.000 states to just 3

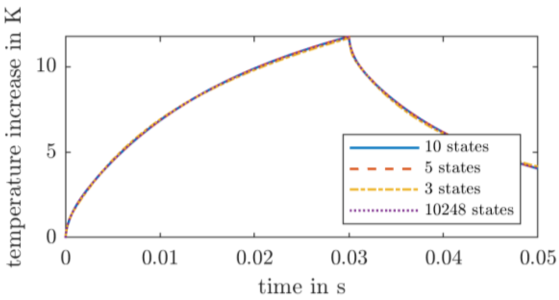

Figure 1: Temperature increase in the RPE during irradiation, calculated with different reduced order models.

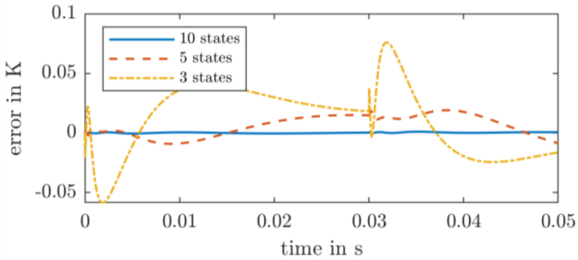

Figure 2: Comparison of error between different reduced order models and the full state model

states. Moreover, smaller errors can be achieved for systems with 5 and 10 states. Similar results are obtained for ΔT_{volume} although the observed errors are higher. In conclusion, the model order can be reduced significantly.

IV. Conclusion

We have introduced a new concept for the temperature-controlled laser photocoagulation based on the PDE of heat diffusion. Some first results of the modelling process have been presented. Using model order reduction based on balanced truncation, the number of states can be reduced substantially. Current work consists of designing suitable observers based on the developed reduced-order models and their subsequent employment in different controller design methods as described afore.

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